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ISTAM Annual Symposium

TECHNICAL PROGRAM

25 December 2011

Tel Aviv University

ISTAM Annual Symposium

25 December 2011

TECHNICAL PROGRAM

Location: Rosenblatt Auditorium, Computer and Software Engineering Building, Tel Aviv University

09:30 – 09:50 Registration and coffee

09:50 – 10:00 *Opening:* MB Rubin

From micro-mechanics to macro-properties: Chairman: K Schulgasser
In honor of Prof. Zvi Hashin receiving the 2012 Benjamin Franklin Medal

10:00 – 10:20 **Z Hashin**. *Composite materials in the Bible*

10:20 – 10:45 **E Altus**, E Rejovitzky, M Kirikov, A Proskura. *Analytical relations for heterogeneous structures*

10:45 – 11:10 **Y Benveniste**, GW Milton. *The effective medium and the average field approximations in matrix-based composites*

11:10 – 11:35 **G deBotton**, G Shmuel, T Oren, Y Goldenberg. *Soft composites attaining the Hashin-Shtrikman bounds at the referential state*

11:35 - 11:45 Break

11:45 – 12:10 **H Chai**. *Tooth enamel as a smart biocomposite for fracture resistance*

12:10 – 12:35 **S Ryvkin**. *Analytical solution for a crack in an open cell foam with periodic microstructure*

12:35 – 13:00 **D Mordehai**. *A combined experimental/computational study of the strength of gold nanoparticles*

13:00 – 13:25 **A Rabinovich**, G Dagan and T Miloh. *Transient heat conduction in a semi-infinite medium with a spherical inhomogeneity*

13:25 – 14:45 Lunch (The registration fee includes lunch)

Afternoon Session Chairman: S Ryvkin

14:45 – 15:10 **I Benichou**, S Givli. *On the structure of the muscle protein titin and its relation to shape memory alloys*

15:10 – 15:35 **Y Shokef** and SA Safran. *Nonlinear elasticity in the interaction of living cells with their mechanical environment*

15:35 – 16:00 **S Vigdergauz**. *An efficient generalization of the equi-stress criterion in optimizing regularly perforated elastic plates*

16:00 – 16:25 **O Shoshani** and O Gottleib. *Vortex-induced vibration and orbital stability of a tethered light sphere in steady fluid flow*

The annual membership fee to ISTAM is 100 NIS. It includes the lunch at the symposium and can be paid during the registration.

All lectures are open to the public free of charge

Composite materials in the Bible

Z Hashin

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Recipient of the 2012 Benjamin Franklin Medal in Mechanical Engineering from The Franklin Institute

The Franklin Institute Awards Program dates back to 1824 and encompasses the Benjamin Franklin Medals in seven disciplines of science and engineering and two Bower Awards – one in Scientific Achievement and one in Business. The list of past Franklin Institute Laureates reads like a Who's Who in 19th, 20th, and 21st century science and engineering, including luminaries Marie and Pierre Curie, Nikola Tesla, Thomas Edison, Max Planck, Orville Wright, Albert Einstein, Marshall Nirenberg, Stephen Hawking, Allen Newell, Ralph Cicerone, Jane Goodall, Don Norman, Steven Squyres and Bill Gates. Recent recipients of the Benjamin Franklin Medal in Mechanical Engineering are: Dean Kamen (2011), D. Brian Spalding (2010), Richard Robbins (2009, Engineering), Merton Flemings (2007), Ray Clough (2006, Civil Engineering), Peter Vail (2005), Roger Bacon (2004), and Charles Thornton (2003, Civil Engineering).

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Analytical relations for heterogeneous structures

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There are many sources of complexity in structural behavior: geometry, kinematic, material nonlinearities and heterogeneity. Analytical solutions are very rare and specific. On the other hand, numerical solutions are limited by insight, generality and computer resources. The method of bounds is an example of analytical or simplified results in spite of the general complexity. In this study, the idea of searching for partial, but analytical relations of a complex problem will be explored by examples concerning heterogeneous structures.

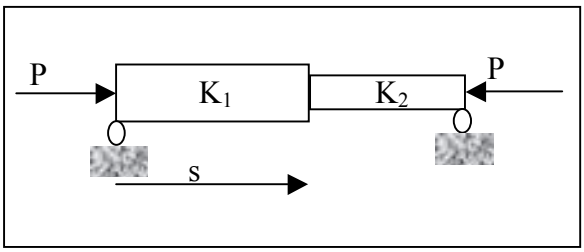

Fig. 1

Example 1: It was found (accidentally) and proven afterwards, that the statistical average buckling load of a simply supported beam composed of two homogeneous parts with random size s (see Fig.1), has a simple analytical solution: $\langle P \rangle = \sqrt{P_1 P_2}$. This is in spite of the fact that each specimen from the ensemble has a numerical solution only.

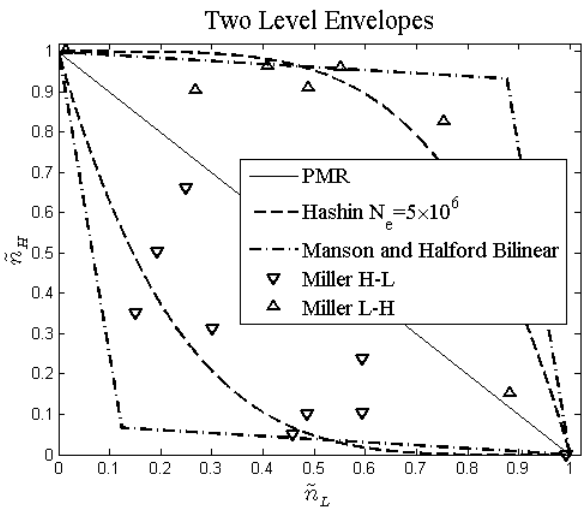

Fig. 2

Example 2: Fatigue models are mostly phenomenological and nonlinear. As a result, their life predictions are achieved numerically. In spite of this limitation, it was found that all fatigue models which are based on a single damage variable, have specific analytical properties, as seen in figure 2: the H-L and L-H graphs are anti-symmetric with respect to the Miner line.

Example 3: An analytical relation for the functional derivative of the buckling load λ with respect to material heterogeneity $D(x_1,x_2)$ of a plate, was found. The fact that this relation is independent of D helped to improve the accuracy and efficiency of an optimization problem of a plate with a hole. The expression

$$\lambda_{D_1} = \frac{m_{ijkl} w_{,x_1^i x_1^j} w_{,x_1^k x_1^l}}{\left(n_{ij} w_{,x^i}\right)_{,x^j} * w}$$

was used for the optimization process shown in Fig. 3.

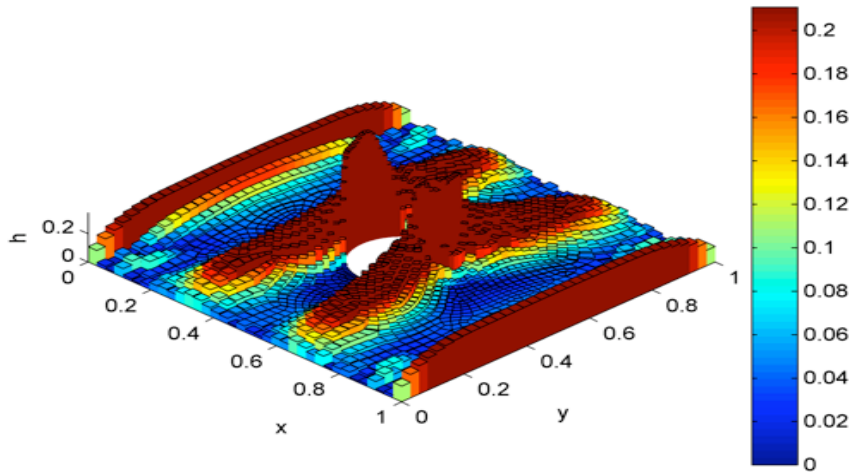

Fig. 3 Optimized thickness distribution of a plate with a hole against buckling.

How to find, evaluate and use, in a systematic way, this analytical partial information for improved solutions and insight of complex problems is an open question .

The effective medium and the average field approximations in matrix-based composites

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The effective medium approximation (EMA) and the average field approximation (AFA), known also in the literature as 'self-consistent' approximations, are two classical micromechanics models for the determination of the effective properties of heterogeneous media. It turns out that the AFA and EMA are often confused with each other, apparently because they produce the same effective property predictions for granular aggregates which consist of spherical or aligned identical ellipsoidal grains of different phases. However, those models are based on different assumptions and produce different results in other circumstances.

Our study reported in [1], [2] and [3] is concerned with the application of the AFA and EMA to matrix-based composites with isotropic constituents, and with the discussion of their predictions vis-à-vis the Hashin-Shtrikman bounds. In matrix-based systems, the AFA makes use of an auxiliary configuration in which a particle of the inclusion phase is embedded in the effective medium, either directly (as in [1]), or after being coated by some matrix material, (as in [2] and [3]) . The EMA, on the other hand, makes always use of the configuration of a matrix-coated particle of the inclusion phase which is embedded in the effective medium (as in [2] and [3]). While due to its realizability property, the EMA always satisfies those bounds, we show that the AFA violates them in several circumstances. Paper [1] is concerned with the AFA which uses the 'direct embedding' configuration and is applied to two-phase systems containing a random distribution of simply-connected arbitrarily shaped particles; it is written in the setting of dielectrics and elasticity. Papers [2] and [3] are concerned with the AFA and EMA which use the 'coated-particle embedding' configuration, and are respectively applied to two-phase and multiphase matrix-based composites; they are written in the setting of dielectric composites containing randomly oriented ellipsoidal particles. The EMA emerges from this study as the one to be recommended among those two classical micromechanics models.

References:

[1] Y. Benveniste, G.W. Milton, 2010. The effective medium and the average field approximations vis-à-vis the Hashin-Shtrikman bounds. I. The self-consistent scheme in matrix-based composites. J. Mech. Phys. Solids 58, 1026-1038 .

[2] Y. Benveniste, G.W. Milton, 2010. The effective medium and the average field approximations vis-à-vis the Hashin-Shtrikman bounds. II. The generalized self-consistent scheme in matrix-based composites. J. Mech. Phys. Solids. 58, 1039-1058.

[3] Y. Benveniste, G.W. Milton, 2011. An effective medium theory for multi-phase matrix-based dielectric composites with randomly oriented ellipsoidal inclusions. Int. J. Engng. Sci. 49, 2-16.

Soft composites attaining the Hashin-Shtrikman bounds at the referential state

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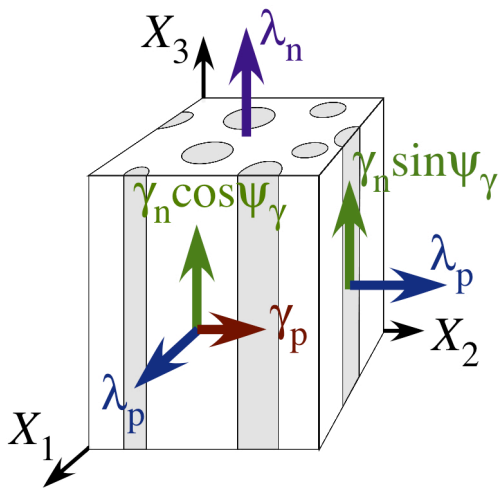
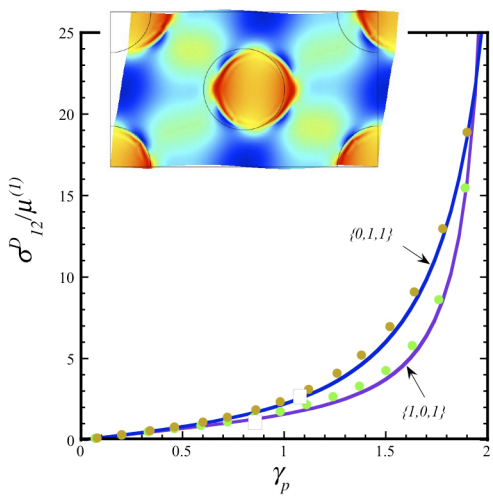
We characterize the macroscopic response of hyperelastic fiber composites in terms of the behaviors of the constituting phases. To this end we make use of a unique representation of the deformation gradient in terms of a set of five transversely isotropic invariants. Physically, these invariants correspond to extension along the fibers, transverse dilatation, out-of-plane shear along the fibers, in-plane shear in the transverse plane and the coupling between the shear modes.

With the aid of this representation, it is demonstrated that there is a class of nonlinear materials for which the exact expression for their macroscopic behavior can be determined analytically. Remarkably, in the limit of infinitesimal elasticity these microstructures attain the well-known Hashin-Shtrikman bounds on the class of transversely isotropic composites. The resulting explicit estimates for the macroscopic stresses developing in composites with neo-Hookean and Gent-like phases are compared with corresponding finite element simulations of periodic composites with hexagonal unit cell. It is demonstrated that the stress-strain curves are in fine agreement with the corresponding numerical results.



A sketch of a fiber composite with the physically motivated TI invariants



Analytical (curves) vs. numerical (marks) predictions of stress-stretch relations

Tooth enamel as a smart biocomposite for fracture resistance

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The enamel coat of mammalian teeth is brittle, yet capable of absorbing considerable deformation and damage from biting and chewing forces under harsh conditions. Enamel is a highly sophisticated composite material integrating a woven, hierarchical rod structure and a cathedral-like geometry to impart unique damage tolerance. This study examines the fundamental nature of enamel resilience by conducting fracture experiments and post-mortem morphological studies on extracted human molar teeth. Several failure modes are identified, including median cracks originating from the contact site and propagating downward, channel cracks propagating upward from the cervical base to the occlusal surface, and chipping failure from sharp contact. The evolution of these cracks is followed using a 3D Finite Element analysis. To follow the internal fracture process some select enamel portions are loaded in tension in a free-standing like configuration. The results reveal interesting information on the interaction between cracks and microstructural entities of the enamel. Key elements in the tooth's resilience appear to be tufts – crack-like intrinsic defects emanating from the enamel-dentin junction, and Hunter-Schreger bands (HSB) – undulated enamel rods that may cross each other (decussation). We found stabilization in the evolution of these defects, by stress shielding from neighbors, as well as by inhibition of ensuing crack extension from prism interweaving (decussation). These features work to slow crack propagation in the enamel and enhance teeth survivability. Finally, we discuss how fracture patterns found in enamel fossils in mammalian teeth can be used to estimate bite force and possibly type of food consumed.

Analytical solution for a crack in an open cell foam with periodic microstructure

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The brittle fracture behavior of an open cell foam is considered. The foam is modeled by an infinite lattice composed of elastic straight-linear beam elements with uniform cross-sections rigidly connected at the nodal points. The beams are parallel to three mutually orthogonal lattice vectors thus producing a microstructure with rectangular parallelepiped cells (Fig. 1). The horizontal faces of the cells are squares, and the cell geometry is completely defined by the shape anisotropy ratio "R=h/l" of the vertical to the horizontal beam elements lengths. A semi-infinite Mode III crack is embedded in the lattice parallel to one of the vertical faces. Consequently, for the considered antiplane deformation each node has three degrees of freedom, namely, vertical displacement and two rotations about the horizontal axes. The analysis method hinges on the discrete Fourier transform which allows to formulate the crack problem by means of the Wiener-Hopf equation. Its solution yields closed form analytical expressions for the forces and displacements at any cross-section, and, in particular at the crack plane.The eigensolution for the traction-free crack faces and K-field remote loading is derived from the solution for the loaded crack by a limiting procedure. The long-wave asymptotes for the displacements and forces observed in the eigensolution on a macroscale correspond to the stress-strain field for the Mode III crack in the homogeneous material with the effective elastic properties.

The fracture analysis of a lattice material allows to follow the discrete origin of the fracture phenomenon and to determine which part of the total energy release observed on a macroscale is related to the unloading of the first element in front of the crack during its break. In the considered problem this value is found to be a monotonically decreasing function of the shape anisotropy ratio. The assumption that crack propagation takes place when the skin stress in the first unbroken element in front of the crack reaches the rupture stress σ_f of the parent material is adopted. The exact analytical expression for the fracture toughness is obtained in the form of a single quadrature. Its dependence upon the material relative density ρ obtained is found to be consistent with the known analytical and experimental results for Mode~I deformation. A parametric study of this expression is carried out. It is found, in particular, that in the case of identical cross-sections of horizontal and vertical elements there is an optimal value of the shape anisotropy ratio "R" maximizing the fracture toughness (Fig.2). This value is appears to be independent of the lattice material relative density.

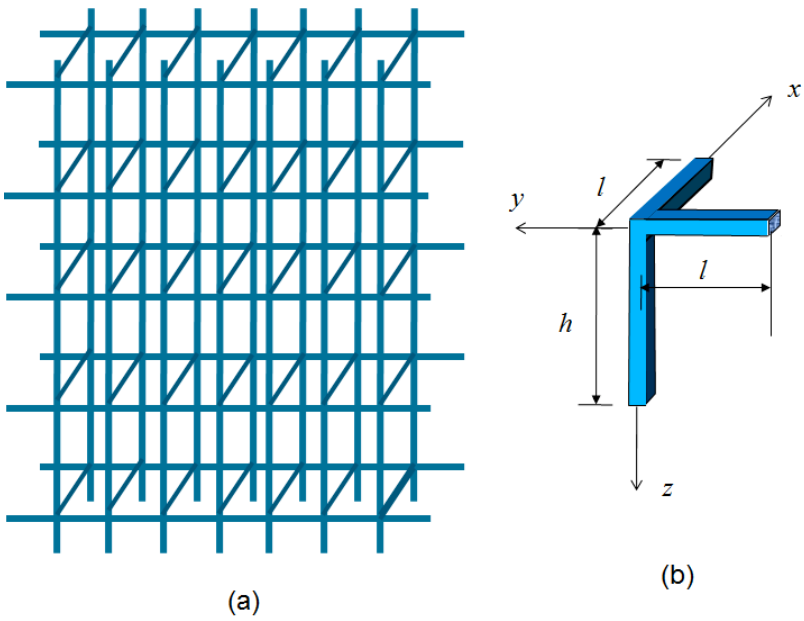

Fig. 1. The 3D beam lattice.

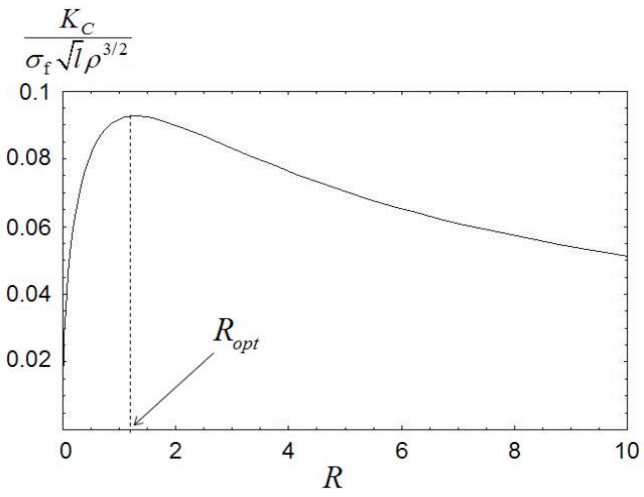

Fig. 2 The normalized fracture toughness vs. shape anisotropy ratio.

Keywords: *open cell foam, beam lattice, discrete Fourier transform, fracture toughness*

Reference:

M. Ryvkin "Analytical Solution for a Mode III Crack in a 3D Beam Lattice" (submitted)

A Combined Experimental/Computational Study of the Strength of Gold Nanoparticles

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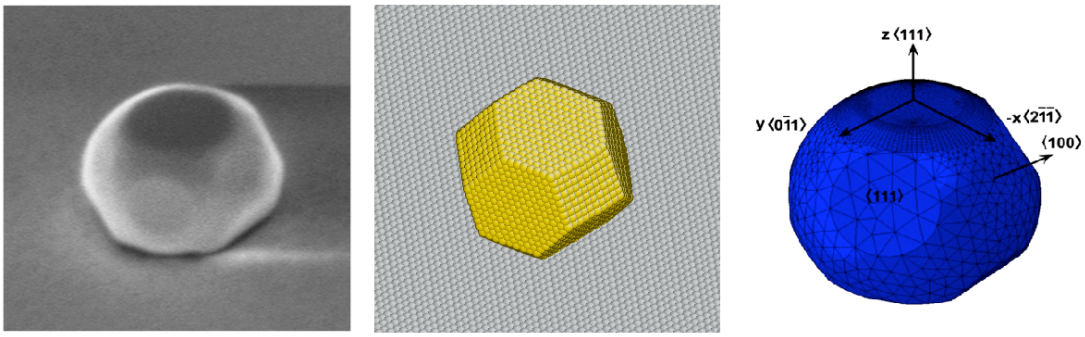
Understanding mechanical properties at the sub-micrometer scale is important, not only for developing a fundamental understanding of plasticity, but also to provide design guidelines for reliable nano- and micro-electromechanical devices. At this scale, the constitutive rules, of which bulk material obeys, break down. In this talk I report a combined experimental/molecular dynamics (MD)/finite element (FE) study of the mechanical properties at the sub-micrometer scale. Faceted Au nanoparticles were created via the agglomeration of a polycrystalline Au film on a sapphire surface (see Figure). Then, nanoparticles with various lateral dimensions (~200nm-1000nm), were deformed under compression. Most of the nanoparticles yielded at compression stresses in the GPa regime with a huge strain burst. Moreover, the compressive stress at the onset of plastic deformation depends strongly on the nanoparticle diameter.

In order to understand the origins of the experimental results, a multiscale study was performed, to relate the microstructural material properties at the nanoscale to the macroscopic deformation of the nanoparticles. MD simulations suggested that the deformation mechanism is controlled by dislocation nucleation on {111} surfaces along <112> directions at the vertices in the periphery of the upper surface, followed by fast dislocation glide toward the surfaces. FE analysis showed that the critical resolved shear stress near the vertices, on the slip planes and along the directions identified in the MD simulations, is comparable with the theoretical shear strength at the yield point found in experiments (in the GPa). In addition, the particles yield strength in the experiments exhibited a strong size effect. To elucidate the size effect observed experimentally, we addressed the simulations again. The elastic properties were found to be size-independent in the MD simulations, but higher compression stresses were needed in order to nucleate dislocations in the smaller particles. Based upon the calculated stress field developing in the particle during compression in the MD and FE simulations, a size-dependent nucleation model was constructed, which rationalizes the size-dependency observed in the experiments.



Experiment MD simulation FEM simulation

Transient Heat conduction in a semi-infinite medium with a spherical inhomogeneity

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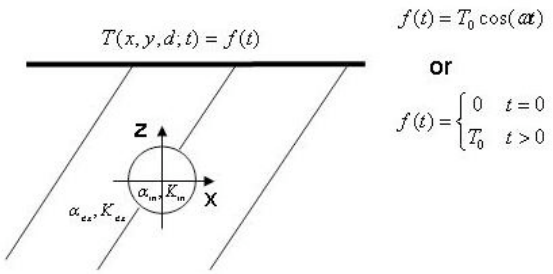
Transient heat conduction in heterogeneous materials is a complex process governed by a non-linear PDE (the heat equation). The present study derives solutions and approximations to a few physical problems of a single spherical inhomogeneity as a first step in an effort to gain insight on the effective or equivalent properties of a medium with an ensemble of spherical inhomogeneities.

We consider a half space, below a planar boundary subjected to time-periodic temperature or alternatively a temperature jump (see figure below). The medium of constant properties (conductivity - K_{ex}, diffusivity - α_{ex}) contains a spherical inclusion of different constant properties (α_{in}, K_{in}). While the solutions for a homogeneous or laminated medium are well documented and provide effective properties were such exist, much less is known for configurations similar to the present one. The applications of primary interest to us are of a geophysical nature, e.g. the impact of heterogeneity upon heat flow through the Earth's crust under periodic (diurnal or annual) temperature variations or under sudden temperature variations (climate change or other deviations from average temperature). However, additional applications regarding heat conduction can also be envisaged as well as other physical processes governed by similar equations and boundary conditions (e.g. electrical conduction, diffusion and flow through porous media).

The problem of a single spherical inhomogeneity is solved. An analytical solution is derived as an infinite series of Bessel functions (in the Laplace domain for the case of a boundary temperature jump), which converges quickly. The general solution is quite involved, yet it can be simplified considerably for asymptotic values of the parameters to yield approximate solutions. Comparison with the exact solution shows that these approximations are accurate for a wide range of parameter values. We illustrate the results and discuss their physical implications for different processes of interest. The simplified solutions can be employed for solving more complex problems consisting of multiple spherical inclusions.



The general setup of the problem: a semi-infinite medium with a spherical inhomogeneity

Keywords: *Spherical inhomogeneity, Effective properties, Transient heat conduction.*

Reference:

A. Rabinovich, G. Dagan and T. Miloh, " Heat conduction in a semi-infinite medium with a spherical inhomogeneity and time-periodic boundary temperature", International Journal of Heat and Mass Transfer (in press).

On the structure of the muscle protein titin and its relation to shape memory alloys

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Titin is a giant protein that functions as a shock absorber in sarcomeres – the basic contractile unit of muscles. When stretched, thermal disturbances are expected to make titin follow the Maxwell path (global minimizer) of its free energy. This path involves neither energy dissipation nor hysteresis. Therefore, a basic question is how does titin release energy so efficiently?

From the mechanical perspective, titin can be conceived as a chain of bi-stable elements (domains) connected in series. Such a system is characterized by a variety of non-trivial equilibrium configurations and inhomogeneous strain distribution. For a prescribed overall strain, several equilibrium configurations are possible. Yet, only one is a global minimizer of the energy.

In this work, we study the mechanical behavior of titin by means of a simple theoretical model of a chain comprised from bi-stable elements, which has been found to be very useful in describing phase-transforming materials. We generalize the model, provide some important insights, and apply it to titin. We show that dissipation depends on both system size and the height of the energy barrier separating equilibrium configurations. In this sense, nature has optimized titin structure in order to function as an efficient shock absorber [1].

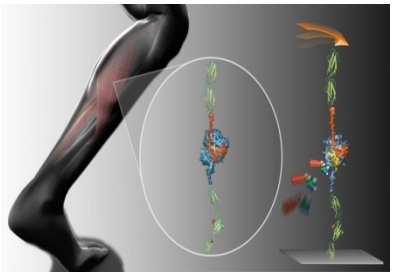

Fig 1: Schematic illustration of Titin muscle.

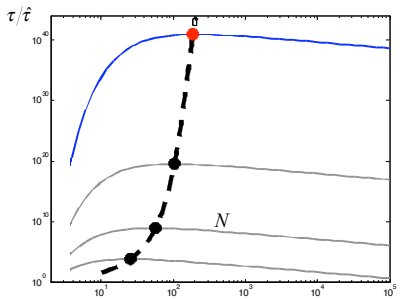

Fig 2: Nature has optimized Titin structure.

Keywords: *Theoretical model, Bio-mechanics, Muscles, Phase transition.*

Reference:

[1] Itamar Benichou and Sefi Givli, 2011, The hidden ingenuity in Titin structure, *Applied Physics Letters* 98(9):091904.

Nonlinear elasticity in the interaction of living cells with their mechanical environment

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The elastic cytoskeleton of biological cells contains molecular motors that produce mechanical forces by which cells attach to and pull on their surroundings. This mechanical interaction is responsible for many aspects of cellular function, from cell spreading and proliferation to stem-cell differentiation and tissue development. Both the cytoskeleton and the extracellular matrix comprise cross-linked, semi-flexible polymeric filaments, and as such they exhibit very nonlinear viscoelastic behavior that includes a power-law stiffening of the elastic moduli with increasing stress [1-4], see Fig. 1. Additional nonlinearity comes from the dependence of the mechanical activity of cells on the rigidity of their environment [5].

Our theoretical work is motivated by traction-force microscopy experiments of cells that adhere to soft gels, in which microbeads are used to measure the deformation field in the gel, and from which the traction forces that the cells exert are deduced. Recent experiments have shown that non-motile cells are dominated by force dipoles, comprised of equal and opposite contractile forces. However, the dependence of strain energy on the total dipole moment exhibits peculiar scaling laws which have not yet been explained [6].

We consider a spherical force dipole (see Fig. 2) in an infinite isotropic, homogeneous, highly nonlinear elastic medium, with constitutive relations derived from a model inspired by [7]. For strong nonlinearity, the differential shear modulus diverges at a finite strain, and we may employ a small strain (but strongly nonlinear) expansion [8,9]. This leads to a nonlinear differential equation for the condition of mechanical equilibrium, from which we extract scaling relations with non-trivial exponents.

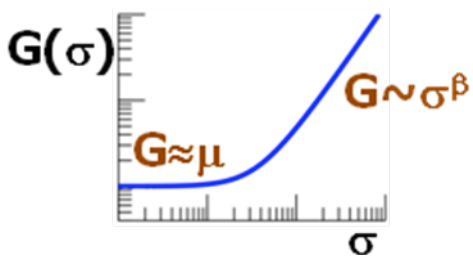

Fig. 1. Differential shear modulus vs stress is initially linear and then crosses over to power-law behavior

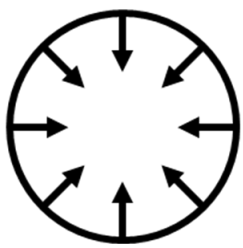

Fig. 2. Spherical force dipole

Keywords: *Mechanobiology, Nonlinear Elasticity, Strain Stiffening, Linearized Strain, Spherical Inclusion.*

References:

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An efficient generalization of the equi-stress criterion in optimizing regularly perforated elastic plates

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Minimization of the hoop stresses variation was recently advanced [1] to form a novel and efficient criterion (V-criterion, for brevity) of the stress state optimization in a statically loaded elastic domain with a finite number of free-form holes. As a radical generalization of the well-known equi-stress principle it offers substantial analytical and numerical advantages over direct minimization of the local stress concentration factor. In particular, the V-criterion averaging nature permits to perform optimization in a wide range of governing parameters at a modest computational cost. Physically, the criterion results in a favorable response of the structure to an external load, with neither local stress concentrations nor underloading of other parts of the boundary. Here we extend the proposed criterion beyond its primary application to the shape optimization in a regular perforated structure with the checkerboard arrangement of identical traction-free holes as illustrated in Fig.1. Due to the bounding rotated square *MNPQ* within the basic cell *ABCD*, the equi-stress holes exist only at small volume fraction values. This is in sharp contrast to the V-criterion whose minimum does exist for *any* geometry.

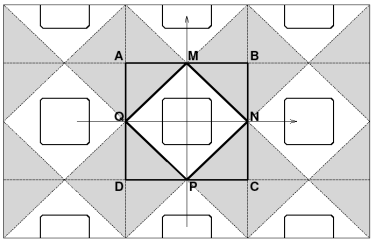

Fig. 1 The problem setup. The basic cell structure is shown by the solid lines

The optimal hole shapes are numerically identified by a three-component approach which includes a genetic algorithm (GA) optimization with an efficient direct solver and with an economic shape parameterization, both formulated in the complex variable terms. The results obtained are partially presented in Fig. 2

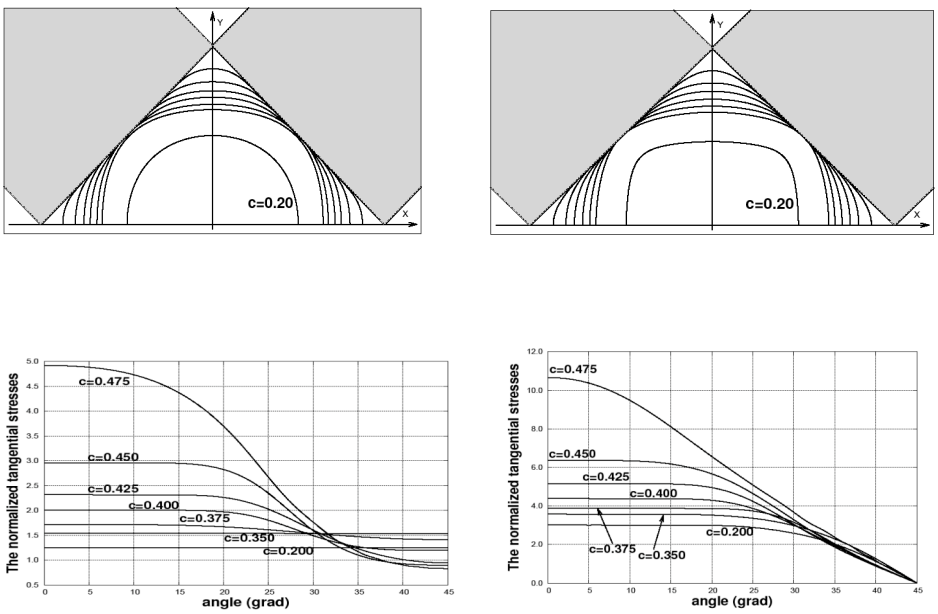

Fig. 2 The V-optimal shapes evolution with the increasing value of c almost up to the percolation limit c=0.5 (the upper row) and the corresponding stress distributions (the bottom row)*

The main conclusions are that the MSV produce a piecewise constant stress distribution which was first introduced for a single hole under remote shear quite differently - as a stationary point of the variation of the

strain energy integral over the solid phase with moving boundaries [2]. The fact that the newly proposed criterion identifies just the same distribution amongst a wide variety of functions with minimum variation strongly counts in its favor.

Keywords: *Plane elasticity problem, checkerboard lattices, shape optimization, Kolosov-Muskhelishvili potentials, hoop stresses, extremal elastic structures, genetic algorithm*

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Vortex-induced vibration and orbital stability of a tethered light sphere in steady fluid flow

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The investigation of vortex-induced vibrations (VIV) of tethered spheres [1] has received a growing amount of attention for over a decade. Examples include cable-buoy ocean systems [2] and aerostats for both surveillance [3] and solar-power applications [4]. These studies have revealed that tethered sphere dynamics include multiple modes of periodic motion [1] and non-stationary response that has been documented for heavy spheres in air [5] and both neutral [6] and heavy [7] spheres in water. However, there is no documentation to-date of a comprehensive theoretical model that enables prediction of the onset of sphere VIV or of its loss of periodicity and consequent classification of non-stationary dynamics. Thus, the objective of this research are to derive, validate, and investigate a dynamical system model for a tethered light sphere in water that captures the VIV stability threshold for self-excited vibration, and to further investigate the orbital stability of the post bifurcation limit-cycle motion.

We formulate a nonlinear dynamical system for a lumped-mass constrained by a visco-elastic tether (Fig. 1) using a Lagrangian formulation [8] that incorporates both and buoyancy and hydrodynamic added mass. The generalized forces in this model include biased and time-dependent hydrodynamic drag, transverse and lift components, and structural damping of a viscoelastic Voight-Kelvin tether deduced from a Rayleigh dissipation function. Motivated by experimentally measured fluctuation of the hydrodynamic forces, we augment the structural system with a nonlinear wake oscillator model that includes time-dependent drag, transverse and lift components. Based on VIV experiments of light spheres in steady flow [9] where the frequency of the in-line response was measured to equal twice that of the transverse direction, we introduce quadratic coupling terms in our wake oscillator model [10]. The resulting strongly nonlinear twelve-order dynamical system reduces for large stiffness, to that of a recently derived spherical pendulum in steady flow [11] that exhibits self-excited limit-cycle motion (Fig. 2) and has been shown via multiple-scale asymptotics, to lose its orbital stability culminating with a quasiperiodic response (Fig.3) near a ($\omega_x : \omega_y : \omega_D : \omega_T$)~(1:1:2:1) internal resonance. Equilibrium analysis of the system with moderate stiffness reveals conditions for a complex ($\omega_x : \omega_y : \omega_z : \omega_D : \omega_T : \omega_L$)~(1:1:2:2:1:1) internal resonance that is analyzed numerically to yield an intricate bifurcation structure incorporating periodic and aperiodic response of the self-excited light sphere in steady flow.

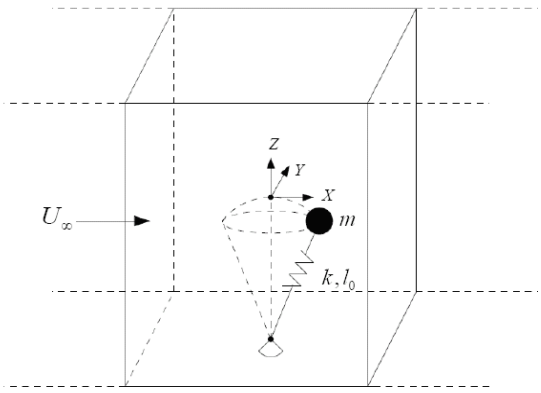

Fig. 1. Elastic tethered sphere definition sketch

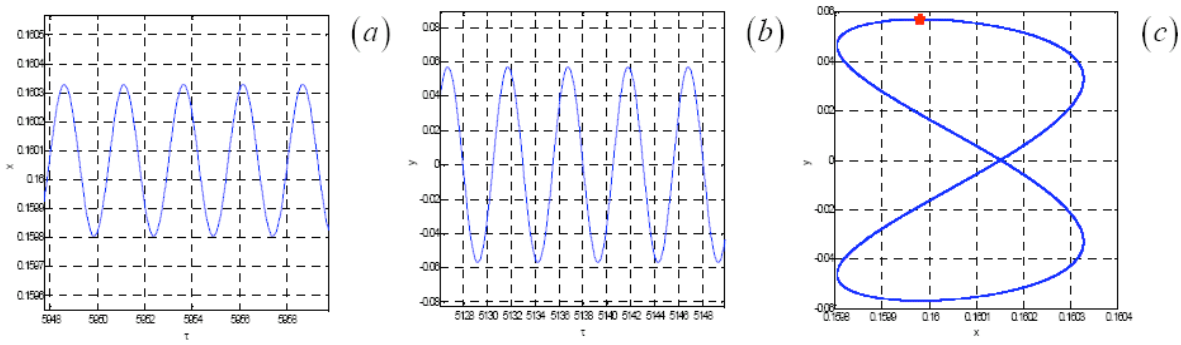

Fig. 2. Periodic VIV- streamwise(a)and transverse(b) displacements time series, and corresponding physical y(x) state-space projection overlaid with its Poincare' point (c).

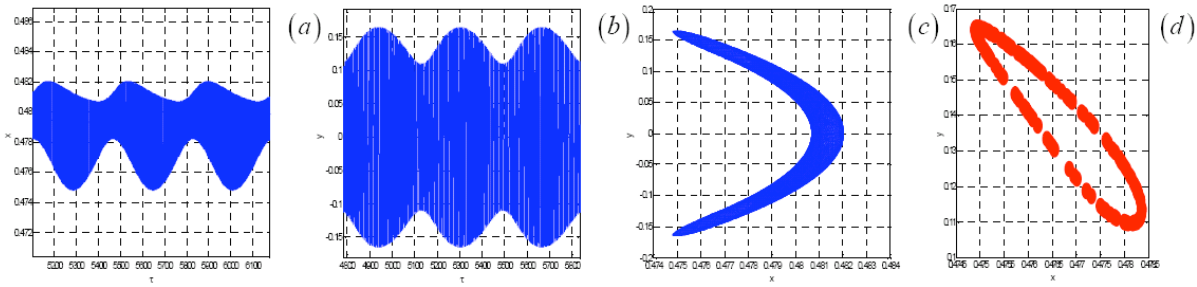

Fig. 3. Quasiperiodic VIV- streamwise(a)and transverse(b) displacements time series, physical y(x) state-space projection(c) and Poincare' map(d).

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