האגוד הישראלי למכניקה עיונית ושימושית

The Israel Society for Theoretical and Applied Mechanics

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ISTAM Annual Symposium

TECHNICAL PROGRAM

13 December 2009

Tel Aviv University

ISTAM Annual Symposium

13 December 2009

TECHNICAL PROGRAM

Location: Rosenblatt Auditorium, Computer and Software Engineering Building, Tel Aviv University

09:30 – 09:50 Registration and coffee

09:50 – 10:00 *Opening:* I Goldhirsch, Tel Aviv University

Morning Session Chairman: S Givli, Technion

10:00 – 10:30 **H Drezner**, Shilo, Dorogoy, Zolotoyabko, Technion. *Nano-scale mapping of mechanical characteristics in nacre layer of mollusk shells*

10:30 – 11:00 **E Faran**, D Shilo, Technion. *Dynamics of twinning processes in active materials at the microscopic scale of individual twins*

11:00 – 11:30 **E Priel**, Z Yosibash. Ben Gurion University. *Analytical and numerical investigation of the response of anisotropic hyper-elastic materials*

11:30 – 12:00 **Y Halevi**, Technion. *Laplace transfer function modeling of flexible structures*

12:00 – 12:30 L Falach, **R Segev**. Ben Gurion University. *On the load capacity of plastic structures*

12:30 – 14:00 Lunch (The registration fee includes lunch)

Afternoon Session Chairman: G Zilman, Tel Aviv University

14:00 – 14:30 **A Liberzon**, Tel Aviv University. *Lagrangian aspects of turbulent flows with additives*

14:30 – 15:00 **Z Kizner**, Govorukhin, Trieling, Nieuwenhuijsen, van Heijst, Bar-Ilan University. *Multipolar vortices: Experimental evidence, explicit solutions, and numerical simulations*

15:00 – 15:30 **E Sharon**, The Hebrew University. *Shaping via Active Deformation of Synthetic and Natural Elastic Sheets*

15:30 – 16:00 **S Givli**, Technion. *General framework for the stability of multi-phase biological membranes*

The annual membership fee to ISTAM is 60 NIS. It includes the lunch at the symposium and can be paid during the registration.

All lectures are open to the public free of charge.

Nano-scale mapping of mechanical characteristics in the nacre layer of mollusk shells

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Nacre, also known as mother-of-pearl, is a hard biogenic composite material found in mollusk shells and composed of ~95% mineral (calcium carbonate) and ~5% protein-rich organic substance. Amazingly, although the both calcite and aragonite polymorphs of calcium carbonate are brittle materials, nacre layers can still sustain significant inelastic deformation and exhibit toughness 2-3 orders of magnitude higher than that of the mineral itself. Numerous studies have been conducted to deeper understand the origin of the superior mechanical properties of the nacre layer. On the contrary to early studies, recent theoretical and experimental findings show that the nacre microstructure alone cannot explain its high toughness.

In order to shed an additional light on the problem, we apply the nanoscale modulus mapping technique for visualizing the 2D-distribution of the mechanical characteristics in the aragonitic nacre layer of *Perna canaliculus* (Green mussel) shells. These shells have very thick nacre layer, which expands practically through the whole shell thickness. Aragonite tablets reveal strong preferred orientation with the *c*-axis of the orthorhombic unit cell, the axis being perpendicular to the inner shell's surface. Specimens for this research were cut along two perpendicular directions in the *a-b*-plane. By mapping the elastic modules we were able to clearly resolve the aragonitic tablets (high elastic modulus) and organic substance (mainly beta-chitin with lower elastic modulus), between them, as presented in figure 1. In further analysis, we compared our experimental data with finite element simulations, which also took into account the tip radius of curvature and the thickness of organic layers, as measured by means of Electron Back Scattered Diffraction (EBSD). Based on this comparison, we extracted the Young modulus of beta-chitin, E = 40 GPa, which is higher than previously evaluated. It is important to stress that the measured elastic modules exhibit gradual changes across the organic/inorganic interfaces within the spatial range which is at least 3 times wider than the thickness of the organic layers.

We explain this phenomenon to be a result of organic macromolecules located within the aragonite tablets, their concentration being gradually increasing when approaching the organic/mineral interface. Thus, the nacre of mollusk shells should be considered as a natural functionally graded material. A behavior of this type is unique to biogenic materials and distinguishes them from synthetic composite materials. Basing on these findings, we suggest a mechanism for massive plastic deformation in the nacre layer, which may explain its superior fracture toughness.

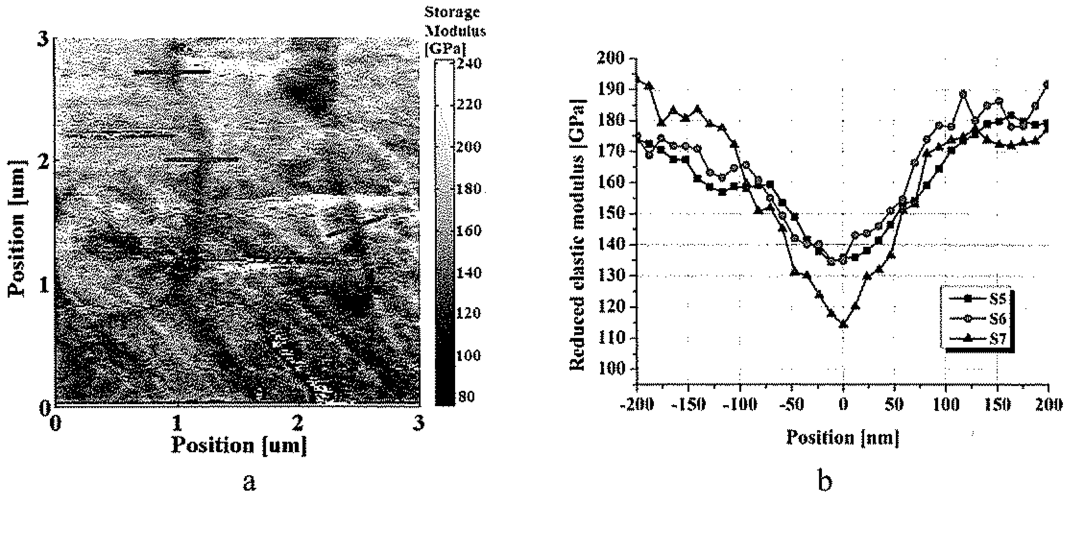

Figure 1: (a) High resolution storage modulus map taken from nacre section in a-b plane and (b) cross-section profiles of the storage modulus across different interfaces. The cross-section lines presented in figure (b) are marked by red lines on figure (a).

Dynamics of twinning processes in active materials at the microscopic scale of individual twins

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Twinning is a shear dominated material transition which plays a significant role in a variety of physical phenomena. In particular, twinning serves as the basic mechanism that enables large stroke actuation in active materials. At the microscopic scale, the twinning transformation involves several sub-processes, including nucleation, forward growth and sideways growth of individual twins. The kinetic relations of these sub-processes (i.e. the rate or velocity as a function of the thermodynamic driving force) are necessary for understanding and modeling the dynamics of twinning transformation.

Here we present an experimental and theoretical study, which focuses on the material response at the microscopic level. A general model for twinning is formulated and is validated experimentally by direct observation of individual twin walls in ferroelectric $BaTiO_3$ and ferromagnetic shape memory alloy NiMnGa single crystals.

Our results demonstrate how kinetic relations of twinning sub-processes are determined by only few parameters, which can be considered as material properties at the atomistic and meso scales. Therefore, extraction of these material parameters will allow predicting the whole twinning kinetic relations under a variety of external conditions. The strong dependence of twinning sub-processes on interaction with defects implies that the velocities of these processes are not a deterministic function of the driving force. Rather, they are controlled by the statistical inhomogeneous distribution of defects. Nevertheless, kinetic relations of a defect-free crystal are still obtained, basing on the maximal velocity values at each driving force, which represent the behavior in a nearly defect free crystal. This is demonstrated in Figure 1, where measured sideways velocities of twin walls in $BaTiO_3$ and NiMnGa are fitted to proposed kinetic relations. The different kinetic laws in the two material types result from differences in materials properties.

In addition, experimental evidence was obtained for forward twin growth faster than the material's speed of sound, with an estimated average velocity close to $\sqrt{2 \cdot C_T}$. This phenomenon is of basic scientific interest as it describes a process that propagates faster than the velocity at which the information about this process travels in the material. From this aspect, inter-sonic motion of crystal defects is analogous to a motion faster than the speed of light, since the energy, the effective mass, and the thickness of the defect core become infinitely large when the defect moves at the speed of sound. On the other hand, in the case of crystal defects, theoretical works do predict the possibility of inter-sonic motion for a variety of defects. However, until our results only inter-sonic cracks have been observed experimentally. Twin forward growth takes place by a collective motion of twin-wall steps, which are linear crystallographic defects similar to dislocations. Therefore, our results strengthen the possibility of inter-sonic dislocation motion and martensitic transformation that have been predicted theoretically, but not observed yet.

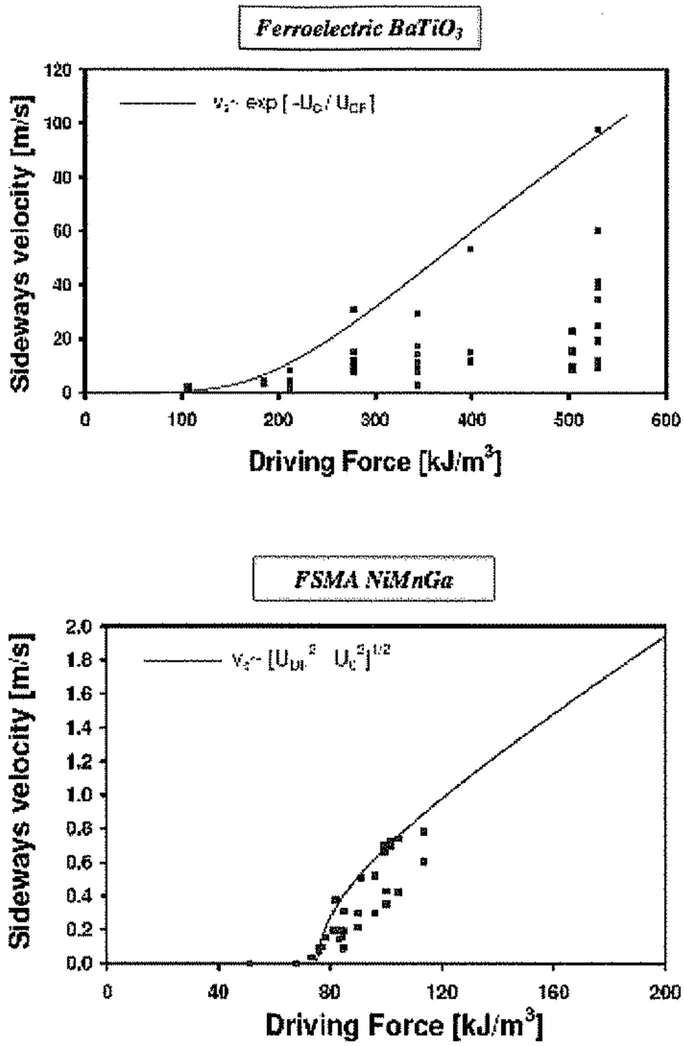

Figure 1: Measured kinetic relations for twin walls' sideways motion in $BaTiO_3$ (top) and NiMnGa (bottom). The different dependencies of wall velocity on driving force are associated with differences between properties of twin walls in the two materials.

Keywords: *Twinning, kinetic relation, active materials*

Analytical and numerical investigation of the response of anisotropic hyper-elastic materials

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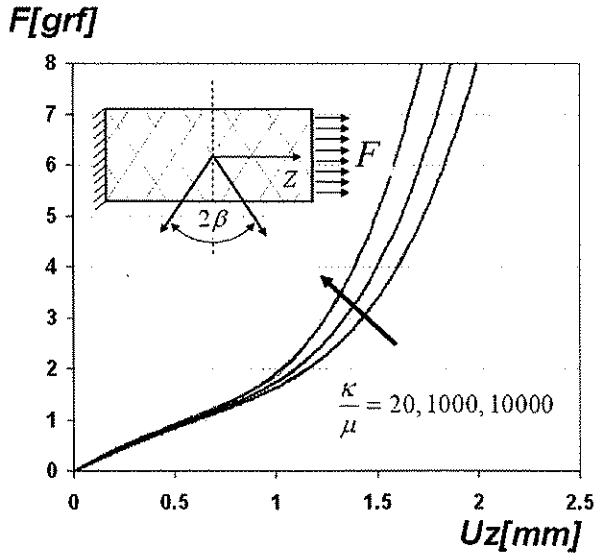
One of the hyper-elastic constitutive models widely accepted for the description of the passive response of arterial walls has been suggested in [1]. It represents a homogenous isotropic incompressible matrix with two families of collagen fibers (arranged in a helix like formation with an angle $\pm\beta$ between the fiber and the circumferential direction). We remove the incompressible constrain in the isotropic hyper-elastic model of [1], addressing a slightly more general strain energy density function of the form:

$$\psi(I_{c1},I_{c3},I_{c4},I_{c6}) = \underbrace{\frac{\mu}{2}(I_{c1}I_{c3}^{-1/3}-3)+\frac{\kappa}{2}(\sqrt{I_{c3}}-1)^2}_{\text{isotopic}} + \underbrace{\frac{k_1}{2k_2}\sum_{i=4,6}[\exp\{k_2(I_{ci}I_{c3}^{-1/3}-1)^2\}-1]}_{\text{anisotropic}}$$

here μ,κ are the shear and bulk modulus of the matrix, k_1,k_2 represent the fiber stiffness. $I_{c1,c3}$ are invariants of the right Cauchy-Green deformation tensor while $I_{c4,c6}$ correspond to stretch in the fiber directions. The mathematical complexity of a non-linear BVP over complex thin-walled domains requires the application of numerical methods such as high-order finite elements.

To verify the accuracy and efficiency of such methods, a set of verification problems with analytic solutions are necessary. Thus, we first present a set of "benchmark" problems for which an analytical solution was derived against which the FE approximations are compared.

Following verification of the high-order FE methods, we will present several investigations on the effect of compressibility, fiber orientation and layer thickness ratio on the passive mechanical response of an artery. For example, in the figure below the force-displacement relationship is presented for an arterial adventitial strip clamped at one end with initial fiber orientation of $\beta=\pm 40^0$. The different curves represent increasing bulk to shear modulus ratio with the last curve (largest ratio) being an incompressible matrix.



Reference:

[1] Holzapfel, G.A. and Gasser, T.C. and Ogden, R.W. 2000, "A new constitutive framework for arterial wall mechanics and a comparative study of material models" *J. Elast* **61**, pp. 1-48.

Laplace transfer function modeling of flexible structures

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Flexible structures are governed by Partial Differential Equations (PDE), and hence have infinite dimension. However, most modeling methods use finite dimension approximations of the system. In the modal approach the infinite dimension appears as an infinite sum of spatial eigenfunctions multiplied by time functions. In practice though, only a finite sum is used. In the popular Finite Element Method (FEM), the finite dimension is achieved by spatial discretization. While finite approximation is practically and even conceptually convenient, some important properties of the system's behavior are lost by it.

This paper takes a different approach. It considers the problem of deriving the accurate, infinite dimension, Laplace transfer function of a system governed by a one-dimensional wave equation. Such systems include strings (e.g. in cranes) and rods in torsion and in tension. The situation is shown schematically in Fig. 1 where the boundary conditions include in general inertia, spring and damper. The system is subjected to a lumped point moment at x_0 and its behavior at a general point x (typically $x=0$ or $x=L$) is of interest.

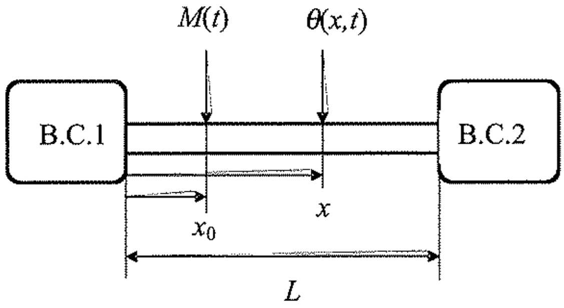

Fig. 1: The flexible system

By means of Laplace transform and some algebraic manipulations it is possible to obtain the transfer function from the applied moment to the angle

$$G(x,x_0,s)=\frac{\theta(x,s)}{M(x_0,s)}=\frac{1}{2\phi s}\cdot\frac{e^{-\beta\tau s}+R_1(s)e^{-(2\eta-\beta)\tau s}+R_2(s)e^{-(2-2\eta+\beta)\tau s}+R_1(s)R_2(s)e^{-(2-\beta)\tau s}}{1-R_1(s)R_2(s)e^{-2\tau s}}$$

where

$$\phi=\frac{GI_p}{c},\quad \tau=\frac{L}{c},\quad \beta=\frac{|x-x_0|}{l},\quad \eta=\frac{\max(x,x_0)}{L},\ R_i(s)=\frac{\phi s-(J_i s^2+D_i s+K_i)}{\phi s+(J_i s^2+D_i s+K_i)}$$

The building blocks of those transfer functions are time delays, representing the wave motion, and low order rational expressions, actually dynamic reflection coefficients, $R_i(s)$, representing the boundary phenomena. The transfer function has thus clear physical meaning from the traveling wave point of view.

The transfer function modeling approach has several theoretical and practical advantages. Simple algebraic investigation of these transfer functions reveals many properties of the flexible structure such as stability, rigid body degrees of freedom, and reciprocity. In the case of conservative boundary conditions this represents a different, and to the best of our knowledge new, way of obtaining well-known results from the modal approach. When the boundary conditions contain dampers, the results presented in this paper do not have counterparts in classical modal analysis. The practical opportunities offered by the transfer function approach are accurate yet simple simulation schemes, exact frequency response for the entire frequency range, analytical solution for the finite time response and identification of dedicated control laws. These particular features and applications were demonstrated in a series of papers.

Keywords: *wave equation, traveling wave, Laplace transform.*

On the load capacity of plastic structures

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Consider a homogeneous isotropic elastic-perfectly plastic body Ω modeled by a bounded open subset of $\Re^3$ having a smooth boundary $\partial\Omega$. The body is assumed to be supported on an open subset Γ_0 of its boundary and let t be the external surface traction acting on the complementary part, Γ_t, of the boundary. Body forces may be included in the analysis but for the sake of simplicity we omit them here. We prove that there exists a maximal positive number C, to which we refer as the *load capacity ratio,* such that the body will not collapse under any external traction field t bounded by $Y_0 C$, where Y_0 is the yield stress. Thus, while the limit analysis factor of the theory of plasticity pertains to a specific distribution of external loading, the load capacity ratio is independent of the distribution of external loading and implies that no collapse will occur for any field t on $\partial\Omega$ as long as $\operatorname{ess\,sup}_{y\in\partial\Omega}|t(y)| < Y_0 C$. Collapse will occur for some t for which the essential supremum of its magnitude over Γ_t is greater than $Y_0 C$. The analysis also allows one to consider subspaces of external loadings, e.g., a collection of loadings acting on a particular subset of the boundary.

The load capacity ratio depends only on the geometry of the body and we prove, see [1], that it is given by

$$\frac{1}{C} = \sup_{w\in LD(\Omega)_D} \frac{\int_{\Gamma_t} |w|\, dA}{\int_\Omega |\varepsilon(w)|\, dV} = \|\gamma_D\|.$$

Here, $LD(\Omega)_D$ is the space of incompressible integrable vector fields w that satisfy the boundary conditions on Γ_0 and for which the corresponding stretchings, or linear strains, $\varepsilon(w)$ are assumed to be integrable. This vector space is equipped with the norm

$$\|w\| = \int_\Omega |\varepsilon(w)|\, dV$$

making it a Banach space. The norm $|\varepsilon(w)|$ on the space of incompressible, or zero-trace, strain matrices should be chosen as the dual of a norm $|\sigma(x)|$ on the space of deviatoric stress matrices induced by the yield criterion, e.g., the Frobenius norm for von-Mises yield criterion. In addition, $\gamma_D : LD(\Omega)_D \to L^1(\partial\Omega, \Re^3)$ is the trace mapping assigning the boundary value $\gamma_D(w)$ to any $w \in LD(\Omega)_D$. It can be shown, see [2], that the trace mapping is well defined. Thus, $1/C$ is the operator norm of the trace mapping.

In [3], we present algorithms that enable the application of this theory for computations of the load capacity ratios of structures. We consider trusses, frames and 2-dimensional finite element models of continuous bodies. As an example, consider the plane strain finite element model shown in Figure 1. It is noted that the support is damaged on the left. The strength of the structure to arbitrary loads applied on the right side is considered. In addition to the evaluation of the load capacity ratio of the structure, we compute on the basis of the theory alone a "worst case loading distribution" to which the structure is most sensitive. Figure 2(a) shows such a worst case loading distribution and Figure 2(b) shows a virtual displacement w that maximizes the expression for $1/C$ as above.

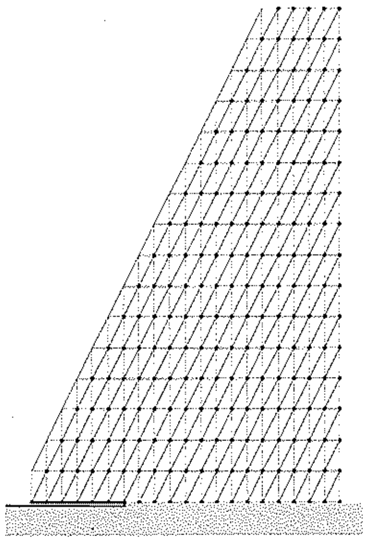
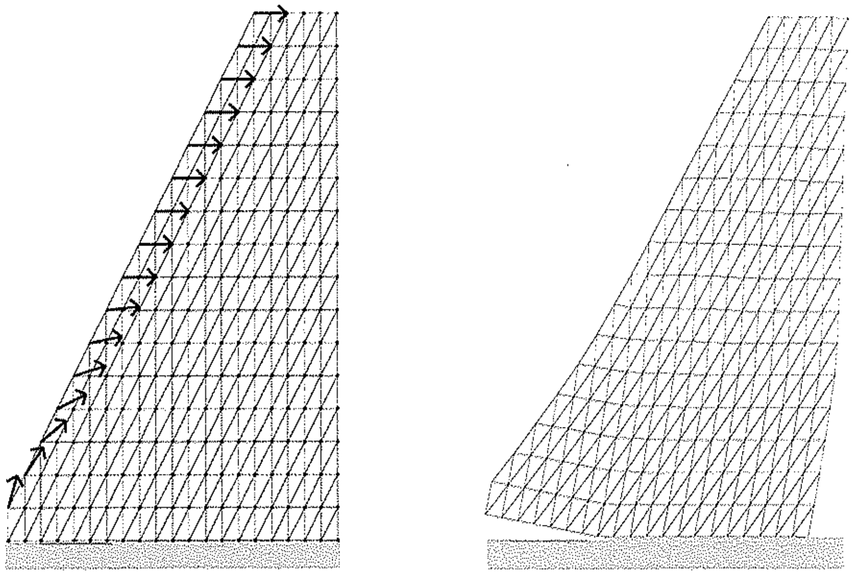


Figure 1. A plane strain finite element model



(a) (b)

Figure 2. (a) Worst load distribution. (b) Maximizing virtual displacement

Keywords: *Continuum Mechanics, optimal stress fields, limit analysis, load capacity.*

References:

[1] Segev, R., Load Capacity of Bodies, *International Journal of Non-Linear Mechanics*, **42**, 250–257 (2007).

[2] Temam, R. (1983), *Problemes Mathematique sen Plasticite*, Bordas, Paris.

[3] Falach, L. & Segev, R., Load Capacity Ratios for Structures, *Computer Methods in Applied Mechanics and Engineering*, in print.

Lagrangian aspects of turbulent flows with additives

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Better understanding of Lagrangian turbulent flow patterns could improve our ability to predict the movement of pollutants, bio-organisms and to quantify turbulent transport and mixing. It was difficult to efficiently, and non-intrusively, obtain Lagrangian data on a large region of turbulent flow, thus very few experimental and numerical methods existed for their effective determination. Recent developments of the three-dimensional particle tracking velocimetry (3D-PTV) enable experimental study of the previously inaccessible Lagrangian properties of turbulent flows. This approach is of special importance for the turbulent flows with additives, such as particles, bubbles or polymer molecules, which are intrinsic Lagrangian objects surrounded by the turbulent fluid. Turbulent flows with additives are characterized by one- and two-way coupling of the fluid and additive phase motions, leading to several unexplained phenomena. An overview of the experimental studies of turbulent flows with particles and dilute polymers solutions in Lagrangian settings will be given, followed by the discussion of the experimental results and the methods of data analysis. In particular, the unique experimental data of the simultaneous measurements of the liquid flow tracers and large, solid particles in Lagrangian settings will be presented, probing directly the two-way coupling of the two phases and to the trends of large particles to cluster in turbulent flows.

Multipolar vortices: Experimental evidence, explicit solutions, and numerical simulations

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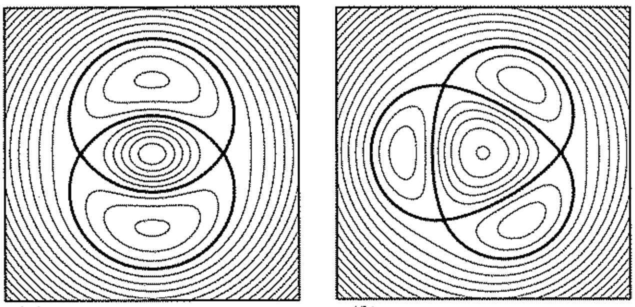
Localized distributed vortical flows are common in ordinary rotating fluids (ocean, atmosphere) and magnetized plasma. We consider vortical multipoles that have been first observed in laboratory experiments (van Heijst & Kloosterziel, 1989). Such a multipole, termed also an $(m+1)$-pole, can be described as an essentially nonlinear two-dimensional stationary solution to the Euler equations in a frame of reference that rotates with the multipole (for mathematical details see Kizner & Khvoles, 2004; Kizner et al. 2007). An $(m+1)$-pole is an ensemble of vortices with the total circulation $2\pi B$ (parameter B can be negative, zero, or positive) that possesses an m-fold symmetry ($m > 1$) and is comprised of a central, core vortex and m satellite vortices surrounding the core (Fig. 1). Fluid parcels in the core and the satellites revolve oppositely, while the multipole as a whole rotates steadily. There is a separatrix with m self-intersection (stagnation) points that demarcates the inner rotational flow from the outer irrotational flow. The behavior of multipoles is studied both numerically and experimentally. A novel setup for laboratory production of multipoles with prescribed parameters in a fast rotating fluid is suggested, it including two coaxial cylinders that can rotate at different azimuthal velocities. Experimentally produced tripoles (m=2), quadrupoles (m=3), pentapoles (m=4), hexapoles (m=5), and heptapoles (m=6) are compared with the mathematical solutions.



(*a*) (*b*)

Fig. 1. Examples of steadily rotating multipoles: co-moving streamfunction at $B = 0$. (*a*) tripole, (*b*) quadrupole

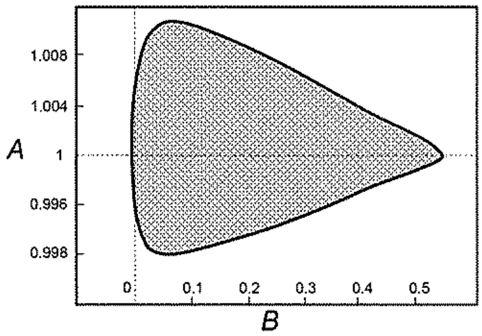


Fig. 2. The quadrupole stability region in the (A, B)-space

The stability of multipoles to different kinds of perturbations is studied numerically with the particle method. Tripoles prove to be substantially stable and multipoles with $m > 3$ unstable. Quadrupoles demonstrate feeble stability in a restricted interval of the B values. As an example, in Fig. 2, the estimated region is shown, in which quadrupoles are stable to strengthening/weakening of one of the satellites. Here B is non-dimensional, scaled with the co-moving streamfunction magnitude assumed at the separatrix; A is the factor by which the vorticity in the satellite of the stationary solution is multiplied.

References

[1] G.J.F. van Heijst and R.C. Kloosterziel. Tripolar vortices in a rotating fluid. Nature **338**, 569 (1989)

[2] Z. Kizner and R. Khvoles. The tripole vortex: Experimental evidence and explicit solutions. Phys. Rev. E **70**, 016307 (2004)

[3] Z. Kizner, R. Khvoles, and J.R.C. McWilliams. Rotating multipoles on the *f*- and *γ*-planes. Phys. Fluids **19**, 016603 (2007)

Shaping via Active Deformation of Synthetic and Natural Elastic Sheets

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Many natural structures are made of soft tissue that undergoes complicated continuous shape transformations due to the distribution of local *active* deformation of their "elements". Examples are the action of the heart, movement of invertebrates and shape development changes during growth. In all these examples, that are very different in their details, smooth distribution of active volume, or orientation, changes, lead to global changes of shape.

Currently, the ability to mimic this mode of shaping in manmade structures is poor. The theoretical difficulties stem from the fact that in general, such bodies have no stress-free configurations, a fact that requires a configuration-free definition of strains. In addition, no reduced theories, such as plate theory exist to such bodies.

The design and construction of actively deforming bodies require manufacturing techniques that do not yet exist, in which local, inducible, volume changes are smoothly varied across the body.

I will present some results of our study of actively deforming thin sheets. We describe the local active "growth" by a local prescribed non-Euclidean "target metric" tensor on the body. This tensor (and not a configuration) is used as a reference, with respect to which we define strains. Using this formalism, we derive a reduced two-dimensional (2D) plate theory for "non-Euclidean plates", plates whose intrinsic 2D metric is non-Euclidean. The 2D theory provides an insight to the principles that govern shaping of non-Euclidean plates. In particular, the mathematical question of embedding 2D surface in 3D Euclidean space is shown to be of special importance for the selection of configurations.

Experimentally, we use environmentally responsive gel sheets to build actively deforming non-Euclidean plates. Controlled, smooth, spatial variation of chemical properties of the gel are used for determining inducible non-Euclidean metrics on the bodies. With this system we study the shaping mechanism and energy scaling in different cases of imposed metrics. Discs with hyperbolic and elliptic target metrics are used as examples for different types of shaping modes.

Finally, we apply this geometrical-mechanical viewpoint to study the evolution of leaf shape during growth. We measure the local growth tensor in wild types and mutants of Arabidopsis and Tobacco leaves. Analysis of these measurements provides quantitative measures of growth, with which we attempt to link between biological activities and the evolution of shape. I will present evidences that in some mutations the observed "complex" leaf shapes do not result from an elaborate growth process. Instead, the mutation causes only a minor change in growth, where the dramatic change in leaf shape results from the mechanical instabilities, induced by this change, and possibly from the feedback of the stress field on local growth.

References

[1] Y. Klein , E. Efrati and E. Sharon (2007) "Shaping of Elastic Sheets by prescription of Non-Euclidean Metrics", *Science* **315**, 1116.

[2] E. Efrati , R. Kupferman and E. Sharon (2009) "Elastic Theory of unconstrained non-Euclidean plates". *Jour. Mech. Phys. Solids,* **57**, 762-775.

[3] E. Efrati , R. Kupferman and E. Sharon , (2009) "Buckling transitions and boundary layers in non-Euclidean plates", *Phys. Rev. E.* , **80**, 016602.

General framework for the stability of multi-phase biological membranes

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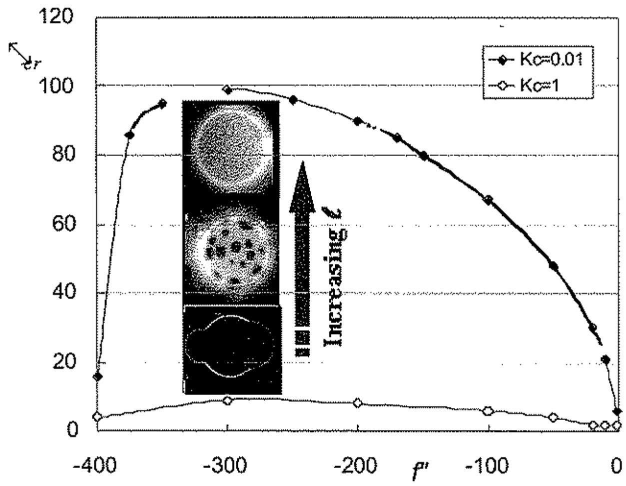
Biological (lipid) membranes are both ubiquitous and fundamental elements of the physics of life. They protect, regulate flow and host many metabolic functions. Biomembranes are made from lipid molecules, proteins, "rigid" cholesterol molecules and other functional molecules. These components differ in their mechanical properties, leading to a complex heterogeneous mechanical structure. Depending on the types of lipids and the functional molecules involved, as well as the external conditions (such as osmotic pressure, external forces, temperature and level of acidity), the biomembrane can remain homogeneous or segregate into different phases. The latter changes the stress distribution in the membrane and either absorb or release energy. Therefore, just like other heterogeneous materials, deformation of the membrane is dictated by composition. However, unlike standard engineering structures, composition is modulated by the membrane shape. Hence, biological membranes are dynamic structures in the sense that their composition and molecular arrangement respond to changes in their environment.

In this talk I will discuss the derivation of equilibrium equations and stability conditions for the general class of biomembranes composed of two phases. These can represent two different lipid phases (e.g. liquid ordered and liquid disordered phases), two different types of lipid molecules, or mobile membrane proteins embedded in a lipid phase. The analysis is based on a generalized Helfrich energy that accounts for the interaction between phases, the coupling between composition and shape, and the non-uniform spatial stretching of the membrane. Further, the use of non-classical differential operators and related integral theorems in conjunction with appropriate composition and mass conserving variations simplify the derivations and avoid the complexity associated with Lagrange Multipliers. Numerical examples demonstrate the importance of the coupling between shape and composition with respect to stability and excited modes. We show that these effects are not intuitive and non-monotonous (for example Fig.1 below). Further, we show that the numerical results are in qualitative accordance with experimental observations.

Fig. 1: The effect of the phase interaction energy, f', on the critical mode, ℓ_{cr}, for a spherical biomembrane. f' can be conceived as a measure of temperature, and ℓ_{cr}^{-1} is proportional to the composition correlation length. In addition higher k_c corresponds to a higher line tension of phase boundaries. Inset: experimental observations of composition landscapes in quasi-spherical membranes by fluorescence microscopy - reproduced from (Baumgart et al. 2003; Veatch and Keller 2003).



Keywords: *biomembrane, heterogeneous, two-phase, stability.*

References:

[1] Baumgart, T., Hess, S. T., and Webb, W. W. (2003). "Imaging coexisting fluid domains in biomembrane models coupling curvature and line tension." *Nature*, **425** (6960), 821-824.

[2] Veatch, S. L., and Keller, S. L. (2003). "Separation of liquid phases in giant vesicles of ternary mixtures of phospholipids and cholesterol." *Biophysical Journal*, **85** (5), 3074-3083.